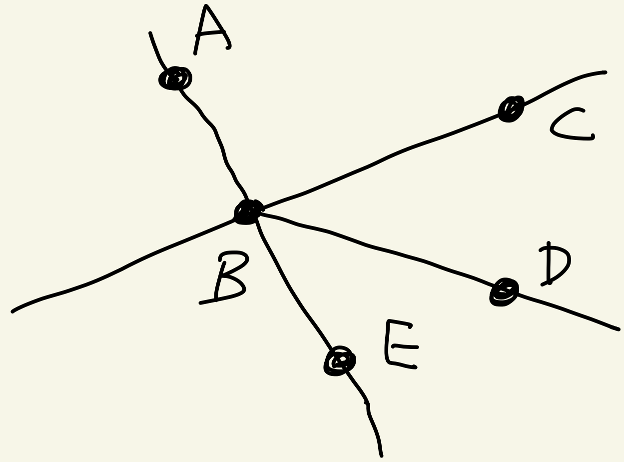
Math 4300
Homework 11
Solutions

1. Suppose A, B, C are non-collinear and C, B, D are non-collinear. Suppose A and D lie on opposite sides of $\overleftrightarrow{BC}$ and $\boxed{m(\angle ABC) + m(\angle CBD) = 180.}$ (*)

We need to show that $A - B - D$.



Let E be such that $A - B - E$.

Then, $\angle ABC$ and $\angle CBE$ form a linear pair.

From the linear pair theorem from class we get that $\angle ABC$ and $\angle CBE$ are supplementary.

That is, $\boxed{m(\angle ABC) + m(\angle CBE) = 180.}$ (**)

Subtracting (*) − (**) gives

$$m(\angle CBD) - m(\angle CBE) = 0.$$

That is, $m(\angle CBD) = m(\angle CBE)$.

Let H be the half-plane that is the side of $\overleftrightarrow{BC}$ that D lives in.

Let $\theta = m(\angle CBD) = m(\angle CBE)$.

By property (ii) of m, we must have that $\overrightarrow{BD} = \overrightarrow{BE}$.

Thus, either $B-D-E$ or $D=E$ or $B-E-D$.

Recall that $A-B-E$.

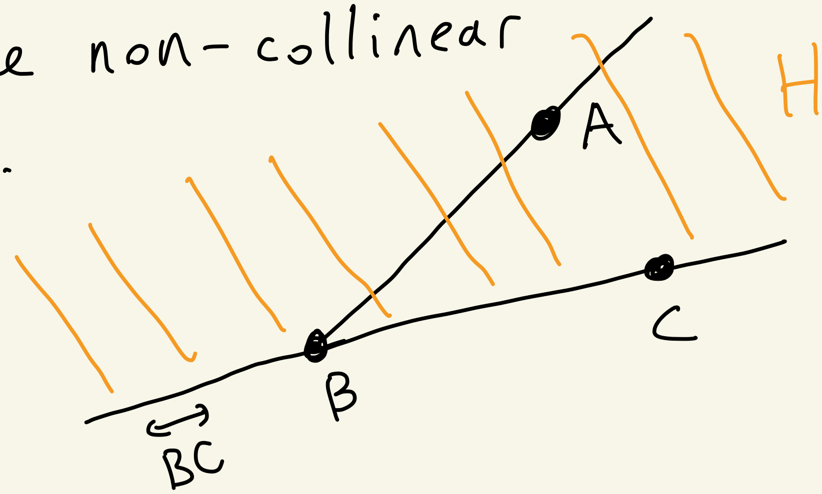
Thus, if $B-D-E$, then $A-B-D-E$.

If $D=E$ then $A-B-D$.

If $B-E-D$, then $A-B-E-D$.

In all cases we get $A-B-D$. $\blacksquare$

② Since A, B, C are non-collinear $\angle ABC$ is an angle.



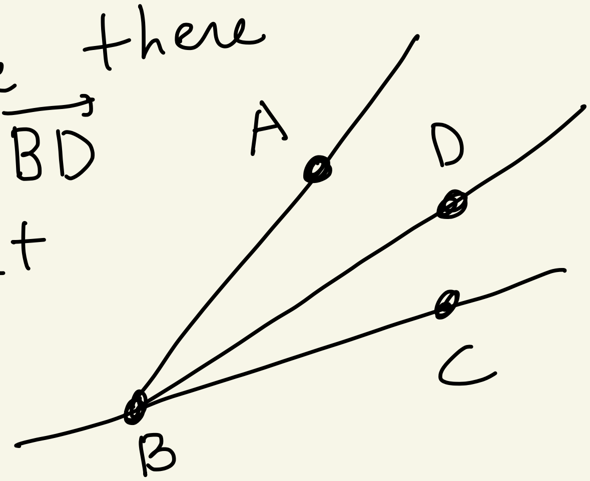
Consider the line $\overleftrightarrow{BC}$.

Since A, B, C are non-collinear, we know that $A \notin \overleftrightarrow{BC}$.

Let H be the halfplane determined by $\overleftrightarrow{BC}$ that contains A.

Let $\theta = m(\angle ABC)$

By (ii) of angle measure there exists a unique ray $\overrightarrow{BD}$ with $D \in H$ such that $m(\angle DBC) = \frac{\theta}{2}$.



Since $D, A \in H$ we know D and A are on the same side of $\overleftrightarrow{BC}$.

Since $m(\angle DBC) = \frac{\theta}{2} < \theta = m(\angle ABC)$ by a theorem in class

We know that $D \in \operatorname{int}(\angle ABC)$.

By (iii) of angle measure, since $D \in \operatorname{int}(\angle ABC)$ we know that

$$m(\angle ABD) + m(\angle DBC) = m(\angle ABC).$$

Thus, $m(\angle ABD) + \frac{\theta}{2} = \theta$.

So, $m(\angle ABD) = \frac{\theta}{2}$.

Thus, $m(\angle ABD) = m(\angle DBC)$. $\blacksquare$